Ultraviolet divergences, renormalization and nonlocality of interactions in quantum field theory.

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Abstract

We discuss the dynamical situation which arises in a local quantum field theory after renormalization. By using the example of the three-dimensional theory of a neutral scalar field interacting through the quartic coupling, we show that after renormalization the dynamics of a theory is governed by a generalized dynamical equation with a nonlocal interaction operator. It is shown that the generalized dynamical equation allows one to formulate this theory in an ultraviolet-finite way.

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The problem of the ultraviolet (UV) divergences is one of the most important problems of quantum field theory (QFT). The renormalized program has not resolved this problem as a whole. For example, regularization of the the scattering matrix gives rise to the situation in which divergent terms automatically appear in the Schrödinger and Tomonago-Schwinger equations. For this reason these equations are only of formal importance to QFT, and the question of what kind of equation governs the dynamics of a theory after renormalization remains unanswered. Since locality has been argued to be the main cause of infinities in QFT, it seems natural to resolve this problem by introducing a nonlocal form factor into the interaction Hamiltonian or Lagrangian of a quantum field theory (see, for example, Ref.[1] and references therein). However, this way of nonlocalization gives rise to several conceptual problems, since such an introduction of a nonlocal form factor is not intrinsically consistent. In fact, the Schrödinger equation is local-in-time, and interaction Hamiltonians describe an instantaneous interaction. In nonrelativistic quantum mechanics processes of instantaneous interaction may be nonlocal in space. But in relativistic quantum theory a local-in-time process must also be local in space. Thus, for nonlocalization to be intrinsically consistent, Hamiltonian dynamics need to be extended to describe the evolution of a quantum system whose dynamics is generated by a nonlocal-in-time interaction. This problem has been solved in Ref.[2], where it has been shown that the current concepts of quantum physics allow such an extension of quantum dynamics. As a consequence of the general postulates of the canonical and Feynman approach to quantum theory a generalized dynamical equation has been derived. Being equivalent to the Schrödinger equation in the case of instantaneous interactions, this equation allows the generalization to the case of nonlocal-in-time interactions. This has been shown [2] to open new possibilities to resolve the problem of the UV divergences in QFT.

Note that the problem of the UV divergences and renormalization is very significant even for describing nucleon dynamics at low energies. In fact, Lagrangians of effective field theories (EFT's) [3] that become very popular in nuclear physics [4] contain terms leading to the UV divergences, and renormalization is needed. A fundamental difficulty in an EFT description of nuclear forces is that are nonperturbative, and one has to renormalize the Schrödinger and Lippmann-Schwinger (LS) equations for describing low-energy nucleon dynamics. However, renormalization gives rise to singular potentials in the case of which these equations have no sense. At the same time, being a unique consequence of the most general principles on which quantum theory is founded, the generalized dynamical equation must be satisfied in all the cases. In Ref.[5] it has been shown that the T matrix obtained in Ref.[6], by renormalizing a toy model of the NN interaction, does not satisfy the LS equation but satisfies the generalized dynamical equation with a nonlocal-in-time interaction operator. This gives reason to suppose that in any theory with the UV divergences after regularization and renormalization the dynamics of the theory is governed by the generalized dynamical equation with such interaction operators. In the present paper the truth of the above assumption is proved by using the example of the supernormalizable theory φ^4_3. We show that after renormalization the dynamics of the theory is governed by the generalized dynamical equation with nonlocal interaction operator, and within a generalized quantum dynamics(GQD) developed in Ref.[2] this theory can be formulated in an ultraviolet-finite way.

In the GQD the following assumptions are used as basic postulates:

(i) The physical state of a system is represented by a vector (properly by a ray) of a Hilbert space.

(ii) An observable A is represented by a Hermitian hypermaximal operator α. The eigenvalues a_r of α give the possible values of A. An eigenvector $|\varphi_r^{(s)}>$ corresponding to the eigenvalue a_r represents a state in which A has the value a_r. If the system is in the state $|\psi>$, the probability P_r of finding the value a_r for A, when a measurement is performed, is given by

$$P_r =<\psi|P_{V_r}|\psi>=\sum_s |<\varphi_r^{(s)}|\psi>|^2,$$

where P_{V_r} is the projection operator on the eigenmanifold V_r corresponding to a_r, and the sum Σ_s is taken over a complete orthonormal set $|\varphi_r^{(s)}>$ (s=1,2,...) of V_r. The state of the system immediately after the observation is described by the vector $P_{V_r}|\psi>$.

These assumptions are the main assumptions on which quantum theory is founded. In the canonical formalism these postulates are used together with the assumption that the time evolution of a state vector is governed by the Schrödinger equation. In the formalism [2] this assumption is not used. Instead the assumptions (i) and (ii) are used together with the following postulate.

(iii) The probability of an event is the absolute square of a complex number called the probability amplitude. The joint probability amplitude of a time-ordered sequence of events is product of the separate probability amplitudes of each of these events. The probability amplitude of an event which can happen in several different ways is a sum of the probability amplitudes for each of these ways.

The statements of the assumption (iii) express the well-known law for the quantum-mechanical probabilities. Within the canonical formalism this law is derived as one of the consequences of the theory. However, in the Feynman formulation of quantum theory [7] this law is directly derived starting from the analysis of the phenomenon of quantum interference, and is used as a basic postulate of the theory.

It is also used the assumption that the time evolution of a quantum system is described by the evolution equation $|\Psi(t) >= U(t,t_0)|\Psi(t_0) >$, where $U(t,t_0)$ is the unitary evolution operator

$$U^{+}(t,t_0)U(t,t_0) = U(t,t_0)U^{+}(t,t_0) = \mathbf{1}, \tag{1}$$

with the group property $U(t,t')U(t',t_0) = U(t,t_0), \quad U(t_0,t_0) = \mathbf{1}$. Here we use the interaction picture. According to the assumption (iii), the probability amplitude of an event which can happen in several different ways is a sum of contributions from each alternative way. In particular, the amplitude $< \psi_2|U(t,t_0)|\psi_1 >$ can be represented as a sum of contributions from all alternative ways of realization of the corresponding evolution process. Dividing these alternatives in different classes, we can then analyze such a probability amplitude in different ways. For example, subprocesses with definite instants of the beginning and end of the interaction in the system can be considered as such alternatives. In this way the amplitude $< \psi_2|U(t,t_0)|\psi_1 >$ can be written in the form [2]

$$< \psi_2|U(t,t_0)|\psi_1 >=< \psi_2|\psi_1 > + \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 < \psi_2|\tilde{S}(t_2,t_1)|\psi_1 >, \tag{2}$$

where $< \psi_2|\tilde{S}(t_2,t_1)|\psi_1 >$ is the probability amplitude that if at time t_1 the system was in the state $|\psi_1 >$, then the interaction in the system will begin at time t_1 and will end at time t_2, and at this time the system will be in the state $|\psi_2 >$. Note that in general $\tilde{S}(t_2,t_1)$ may be only an operator-valued generalized function of t_1 and t_2 [2], since only $U(t,t_0) = \mathbf{1} + \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \tilde{S}(t_2,t_1)$ must be an operator on the Hilbert space. Nevertheless, it is convenient to call $\tilde{S}(t_2,t_1)$ an "operator", using this word in generalized sense. In the case of an isolated system the operator $\tilde{S}(t_2,t_1)$ can be represented in the form $\tilde{S}(t_2,t_1) = exp(iH_0t_2)\tilde{T}(t_2-t_1)exp(-iH_0t_1)$, H_0 being the free Hamiltonian [2].

As has been shown in Ref.[2], for the evolution operator $U(t,t_0)$ given by (2) to be unitary for any times t_0 and t, the operator $\tilde{S}(t_2,t_1)$ must satisfy the following equation:

$$(t_2-t_1)\tilde{S}(t_2,t_1) = \int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 (t_4-t_3)\tilde{S}(t_2,t_4)\tilde{S}(t_3,t_1). \tag{3}$$

This equation allows one to obtain the operators $\tilde{S}(t_2,t_1)$ for any t_1 and t_2, if the operators $\tilde{S}(t'_2,t'_1)$ corresponding to infinitesimal duration times $\tau = t'_2 - t'_1$ of interaction are known. It is natural to assume that most of the contribution to the evolution operator in the limit $t_2 \rightarrow t_1$ comes from the processes associated with the fundamental interaction in the system under study. Denoting this contribution by $H_{int}(t_2,t_1)$, we can write

$$\tilde{S}(t_2,t_1) \underset{t_2\rightarrow t_1}{\longrightarrow} H_{int}(t_2,t_1) + o(\tau^{\epsilon}), \tag{4}$$

where $\tau = t_2 - t_1$. The parameter ε is determined by demanding that $H_{int}(t_2,t_1)$ must be so close to the solution of Eq.(3) in the limit $t_2 \rightarrow t_1$ that this equation has a unique solution having the behavior (4) near the point $t_2 = t_1$.Thus this operator must satisfy the condition

$$(t_2-t_1)H_{int}(t_2,t_1) \underset{t_2\rightarrow t_1}{\longrightarrow} \int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 (t_4-t_3) H_{int}(t_2,t_4) H_{int}(t_3,t_1) + o(\tau^{\epsilon+1}). \tag{5}$$

Note that the value of the parameter ϵ depends on the form of the operator $H_{int}(t_2,t_1)$. Since $\tilde{S}(t_2,t_1)$ and $H_{int}(t_2,t_1)$ are only operator-valued distributions, the mathematical meaning of the conditions (4) and (5) needs to be clarified. We will assume that the condition (5) means that $<\Psi_2|\int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \tilde{S}(t_2,t_1)|\Psi_1> \underset{t\rightarrow t_0}{\longrightarrow} <\Psi_2|\int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 H_{int}(t_2,t_1)|\Psi_1> + o(\tau^{\epsilon+2})$, for any vectors $|\Psi_1>$ and $|\Psi_2>$ of the Hilbert space. The condition (6) has to be considered in the same sense.

Within the GQD the operator $H_{int}(t_2,t_1)$ plays the role which the interaction Hamiltonian plays in the ordinary formulation of quantum theory: It generates the dynamics of a system. Being a generalization of the interaction Hamiltonian, this operator is called the generalized interaction operator. If $H_{int}(t_2,t_1)$ is specified, Eq.(3) allows one to find the operator $\tilde{S}(t_2,t_1)$. Formula (2) can then be used to construct the evolution operator $U(t,t_0)$ and accordingly the state vector $|\psi(t)> = |\psi(t_0)> + \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \tilde{S}(t_2,t_1)|\psi(t_0)>$ at any time t. Thus Eq.(3) can be regarded as an equation of motion for states of a quantum system. It should be noted that Eq.(3) is written only in terms of the operators $\tilde{S}(t_2,t_1)$, and does not contain operators describing the interaction in a quantum system. It is a relation for $\tilde{S}(t_2,t_1)$ which are the contributions to the evolution operator from the processes with defined instants of the beginning and end of the interaction in the system. This relation is a unique consequence of the unitarity condition (1) and the representation (2) expressing the Feynman superposition principle (the assumption (iii)). For this reason the relation (3) must be satisfied in all the cases. A remarkable feature of this fundamental relation is that it works as a recurrence relation, and to construct the evolution operator it is enough to now the contributions to this operator from the processes with infinitesimal duration times of interaction, i.e. from the processes of a fundamental interaction in the system. This makes it possible to use the fundamental relation (3) as a dynamical equation. Its form does not depend on the specific features of the interaction (the Schrödinger equation, for example, contains the interaction Hamiltonian). Since Eq.(3) must be satisfied in all the cases, it can be considered as the most general dynamical equation consistent with the current concepts of quantum theory. As has been shown in Ref.[2], the dynamics governed by Eq.(3) is equivalent to the Hamiltonian dynamics in the case where the generalized interaction operator is of the form

$$H_{int}(t_2, t_1) = -2i\delta(t_2 - t_1)H_I(t_1), \tag{6}$$

$H_I(t_1)$ being the interaction Hamiltonian in the interaction picture. In this case the evolution operator given by (2) satisfies the Schrödinger equation. The delta function $\delta(\tau)$ in (6) emphasizes the fact that in this case the fundamental interaction is instantaneous. At the same time, Eq.(3) permits the generalization to the case where the interaction generating the dynamics of a system is nonlocal in time [2]. In Ref.[8] this point was demonstrated on exactly solvable models. It should be noted that form of the generalized interaction operator cannot be arbitrary, since it must satisfy the condition (5). As has been shown [2,8], there is one-to-one correspondence between nonlocality of interaction and the UV behavior of the matrix elements of the evolution operator as a function of momenta of particles: The interaction operator can be nonlocal in time only in the case where this behavior is "bad", i.e. in a local theory it results in UV divergences.

The above gives reason to expect that after renormalization the dynamics of a theory does not governed by the Schrödinger equation should mean that interaction in the renormalized theory is nonlocal-in-time. This has been illustrated [5] on a toy model of the NN interaction [6]. Let us now consider this problem by using the example of the three-dimensional theory of a neutral scalar field interacting through the φ^4 coupling. The Hamiltonian of the theory, with a spatial cutoff, is of the form

$$H(g) = H_0 + H_I(g), \tag{7}$$

where $H_I(g) = \int d^2x g(\mathbf{x}) : \varphi^4(\mathbf{x}, t = 0) :$, $\varphi(x)$ is the field with mass m, and $::$ denotes the normal ordering. As is well known, introducing a spatial cutoff is needed because of the problems associated with the Haag's theorem. Let g be a $C_0^\infty(R^2)$ function, $0 \leq g \leq 1$.

Since the use of the local interaction Hamiltonian density $\mathcal{H}_I(x, g) = g : \varphi^4(x) :$ leads to the UV divergences, some regularization procedure is also needed. Let the regularized interaction Hamiltonian be of the form

$$H_I^{(\Lambda)}(g) = \int d^2x \mathcal{H}_I(\mathbf{x}, t=0; g, \Lambda)$$

with the interaction Hamiltonian density $\mathcal{H}_I(\mathbf{x}, g, \Lambda) = g : \varphi_h^4(x) :$, where $\varphi_h(\mathbf{x}, t) = \int d\mathbf{x}' h_\Lambda(\mathbf{x}-\mathbf{x}')\varphi(\mathbf{x}', t)$. Here $h_\Lambda(\mathbf{x})$ is a real function such that $h_\Lambda(\mathbf{x}-\mathbf{x}') \underset{\Lambda\to\infty}{\longrightarrow} \delta(\mathbf{x}-\mathbf{x}')$, and $< 0|\varphi_\Lambda(\mathbf{x}, t)\varphi_\Lambda(\mathbf{x}', t)|0 >$ is bounded at $\mathbf{x} = \mathbf{x}'$. In order that after letting Λ to infinity the theory be finite, we have to complement the interaction Hamiltonian in the interaction picture $H_I^{(\Lambda)}(t; g)$ by the renormalization counterterms

$$H_I^{(\Lambda)}(t; g) \rightarrow H_I^{(r)}(t; g, \Lambda) = \int d^2x \mathcal{H}_I^{(r)}(\mathbf{x}, t; g, \Lambda)$$

with $\mathcal{H}_I^{(r)}(x; g, \Lambda) = \mathcal{H}_I^{(\Lambda)}(x; g) - \frac{1}{2}\delta m^2(\Lambda) : \varphi^2(x) : -E(\Lambda)$. The counterterms $E(\Lambda)$ and $\delta m(\Lambda)$ are the contributions to the ground state energy and to the rest mass of a single particle respectively. Since the operator $H_I^{(r)}(t; g, \Lambda)$ is self adjoint and bounded from below on Fock space [9], within perturbation theory one can use the Dyson expansion for constructing the evolution operator

$$U(t, t_0) = \mathbf{1} + \sum_{n=1}^{\infty}(-i)^n \int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t_{n-1}} dt_n H_I(t_1)\cdots H_I(t_n) =$$

$$= \mathbf{1} + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} dt_n T\left(H_I(t_1)\cdots H_I(t_n)\right), \qquad (8)$$

where T denotes the time-ordered product. At the same time we can rewrite Eq.(8) in the form

$$U(t, t_0) = \mathbf{1} - \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \left(2i\delta(t_2 - t_1)H_I(t_1) + H_I(t_2)U(t_2, t_1)H_I(t_1)\right). \quad (9)$$

Thus, within Hamiltonian dynamics, the evolution operator can be represented in the form (2) with the following operator $\tilde{S}(t', t)$:

$$\tilde{S}(t', t) = -2i\delta(t'-t)H_I(t) +$$

$$+ \sum_{n=0}^{\infty}(-i)^{n+2} \int_{t}^{t'} dt_1 \cdots \int_{t}^{t_{n-1}} dt_n H_I(t')H_I(t_1)\cdots H_I(t_n)H_I(t). \qquad (10)$$

It is not difficult to verify that (10) is the solution of Eq.(3) with the boundary condition given by (4) and (6). This means that in the case of the generalized interaction operators of the form (6), the perturbative solution of Eq.(3) results in the Dyson expansion that within Hamiltonian formalism represents the formal solution of the Schrödinger equation.

By using (2) and (10), we can construct the evolution operator corresponding to the interaction Hamiltonian $H_I^{(r)}(t)$

$$U_{r,\Lambda}(t,t_0;g) = \mathbf{1} + \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \tilde{S}_{r,\Lambda}(t_2,t_1;g),$$

where

$$\tilde{S}_{r,\Lambda}(t_2,t_1;g) = -2i\delta(t_2-t_1)H_I^{(r)}(t_1) + \tilde{S}'_{r,\Lambda}(t_2,t_1;g), \tag{11}$$

$$\tilde{S}'_{r,\Lambda}(t',t;g) = \sum_{n=0}^{\infty}(-i)^{n+2}\int_t^{t'} dt_1 \cdots \int_t^{t_{n-1}} dt_n H_I^{(r)}(t';g,\Lambda)\times$$
$$\times H_I^{(r)}(t_1;g,\Lambda)\cdots H_I^{(r)}(t_n;g,\Lambda)H_I^{(r)}(t;g,\Lambda). \tag{12}$$

Then taking Λ to infinity, for the evolution operator and the S matrix, we get

$$U(t,t_0;g) = \lim_{\Lambda\to\infty} U_{r,\Lambda}(t,t_0;g) = \mathbf{1} + \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \tilde{S}_{ren}(t_2,t_1;g), \tag{13}$$

$$S(g) = \mathbf{1} + \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{t_2} dt_1 \tilde{S}_{ren}(t_2,t_1;g),$$

with

$$\tilde{S}_{ren}(t_2,t_1;g) = \lim_{\Lambda\to\infty} \tilde{S}_{r,\Lambda}(t_2,t_1;g). \tag{14}$$

Since the operators $\tilde{S}_{r,\Lambda}(t_2,t_1;g)$ satisfy Eq.(3) for any Λ, the operator $\tilde{S}_{ren}(t_2,t_1;g)$ should also satisfy this equation. Thus $\tilde{S}_{r,\Lambda}(t_2,t_1;g)$ given by (11) is a solution of the dynamical equation (3). Let us now find the generalized interaction operator to which this solution corresponds. Representing the operator $\tilde{S}(t_2,t_1)$ in the form $\tilde{S}(t_2,t_1) = H_{int}(t_2,t_1) + \tilde{S}_1(t_2,t_1)$, dynamical equation (3) can be rewritten as the following equation for the operator $\tilde{S}_1(t_2,t_1)$:

$$(t_2-t_1)\tilde{S}_1(t_2,t_1) = (t_2-t_1)F(t_2,t_1) + \int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 (t_4-t_3)\times$$
$$\times\left(H_{int}(t_2,t_4)\tilde{S}_1(t_3,t_1) + \tilde{S}_1(t_2,t_4)H_{int}(t_3,t_1) + \tilde{S}_1(t_2,t_4)\tilde{S}_1(t_3,t_1)\right), \tag{15}$$

with

$$F(t_2,t_1) = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 (t_4-t_3) H_{int}(t_2,t_4)H_{int}(t_3,t_1) - H_{int}(t_2,t_1).$$

Note that this equation determines the operator $\tilde{S}_1(t_2,t_1)$ only up to some quasilocal operators $\Delta(t_2,t_1)$, i.e., operators which do not zero only for $t_2=t$, satisfying the condition $(t_2-t_1)\Delta(t_2,t_1)=0$. However, the operator $\tilde{S}_1(t_2,t_1)$, by the definition of the operator $H_{int}(t_2,t_1)$, is less singular at $t_2=t_1$ than $H_{int}(t_2,t_1)$, and therefore cannot contain the quasilocal operators. For Eq.(15) to have a unique solution, the operator $H_{int}(t_2,t_1)$ must be close enough to the corresponding solution. This means that $<\Psi_2|\int_{t_0}^t dt_2\int_{t_0}^{t_2}dt_1F(t_2,t_1)|\Psi_2>$ must rapidly enough tend to zero as $t\rightarrow t_0$ for any vectors $|\Psi_1>$ and $|\Psi_2>$. In the case of Hamiltonian dynamics, when $H_{int}(t_2,t_1)$ is of the form (6),

$$F(t_2,t_1)=(-i)^2H_I(t_2)H_I(t_1).$$

In this case

$$<\Psi_2|\int_{t_0}^t dt_2\int_{t_0}^{t_2}dt_1F(t_2,t_1)|\Psi_1>=o(\tau^2)$$

for $\tau\rightarrow 0$, provided $H_I(t)$ is a self adjoint operator on the Hilbert space of physical states. It is easy to verify that the iterative solution of Eq.(15) with $H_{int}(t_2,t_1)=-2i\delta(t_2-t_1)H_I(t_1)$ and $F(t_2,t_1)=(-i)^2H_I(t_2)H_I(t_1)$ yields the expression (10) for the operator $\tilde{S}(t_2,t_1)$. Note that one can include the term $(-i)^2H_I(t_2)H_I(t_1)$ into the generalized interaction operator. The operator $H_{int}(t_2,t_1)$ constructed in this way is closer to the solution of Eq.(3) than $-2i\delta(t_2-t_1)H_I(t_1)$: In this case we start with the generalized interaction operator, being the solution of Eq.(3) in the second order approximation. Obviously, the generalized interaction operators $-2i\delta(t_2-t_1)H_I(t_1)$ and $-2i\delta(t_2-t_1)H_I(t_1)+(-i)^2H_I(t_2)H_I(t_1)$ are dynamically equivalent because they yield the same dynamics. However, the operator $-2i\delta(t_2-t_1)H_I(t_1)$ contains the all needed dynamical information, and it is natural to use this operator as the generalized interaction operator generating Hamiltonian dynamics.

Let us come back to our model now. The production $(-i)^2H_I(t_2;g)H_I(t_1;g)$ has the nonintegrable singularity at the point $t_2=t_1$, and as a consequence is not an operator valued distribution, i.e. $\int_{t_0}^t dt_2\int_{t_0}^{t_2}dt_1(-i)^2H_I(t_2;g)H_I(t_1;g)$ is not an operator on Fock space. For this reason we have to introduce the momentum cutoff that should be removed after renormalization. This means that in Eq.(15) we have to use the generalized interaction operator $H_{int}(t_2,t_1)=-2i\delta(t_2-t_1)H_I^{(r)}(t_1;g)$. In this case the operator $F(t_2,t_1)$ is of the form $F(t_2,t_1)=(-i)^2H_I^{(r)}(t_2;g)H_I^{(r)}(t_1;g)$. However we cannot let $\Lambda\rightarrow\infty$ in Eq.(15) with such operators $H_{int}(t_2,t_1)$ and $F(t_2,t_1)$, since the counterterms $i\delta(t_2-t_1)\delta m^2(\Lambda)\int d^2xg(\mathbf{x}):\varphi^2(\mathbf{x},t_1):$ and $2i\delta(t_2-t_1)E(\Lambda)\int d^2xg(\mathbf{x})$, that have to eliminate the divergences of the operator $\lim\limits_{\Lambda\rightarrow\infty}(-i)^2H_I^{(r)}(t_2;g)H_I^{(r)}(t_1;g)$ in $F(t_2,t_1)$, are contained in the operator $H_{int}(t_2,t_1)$. On the other hand, we can take the limit $\Lambda\rightarrow\infty$ in Eq.(15), if we start with the interaction operator

$$H_{int}(t_2,t_1)=-2i\delta(t_2-t_1)H_I^{(r)}(t_1;g)+(-i)^2H_I^{(\Lambda)}(t_2;g)H_I^{(\Lambda)}(t_1;g)$$

that is dynamically equivalent to the operator $-2i\delta(t_2-t_1)H_I^{(r)}(t_1;g)$ for finite Λ. In fact, in this case the generalized interaction operator $H_{int}^{(\Lambda)}(t_2,t_1)$ contains the term $(-i)^2H_I^{(\Lambda)}(t_2;g)H_I^{(\Lambda)}(t_1;g)$ together with the corresponding counterterms $i\delta(t_2-t_1)\delta m^2(\Lambda)\int d^2xg(\mathbf{x}):\varphi^2(\mathbf{x},t_1):$ and $2i\delta(t_2-t_1)E(\Lambda)\int d^2xg(\mathbf{x})$. However, since in the model under study $<0|H_I(t_1;g)H_I(t_2;g)H_I(t_3;g)|0>$ has the nonintegrable singularity at the point $t_1=t_2=t_3$, the counterterm $E(\Lambda)$ is of the form $E(\Lambda)=E_2(\Lambda)+E_3(\Lambda)$, where $E_2(\Lambda)$ and $E_3(\Lambda)$ are the vacuum energy counterterms in the second and third orders respectively. For this reason we have to include the term $\int_{t_1}^{t_2}dt(-i)^3<0|H_I^{(\Lambda)}(t_2;g)H_I^{(\Lambda)}(t;g)H_I^{(\Lambda)}(t_1;g)|0>$ into the generalized interaction operator. After this we can let $\Lambda\to\infty$. In this way we get the generalized interaction operator of the theory

$$H_{int}(t_2,t_1)=-2i\delta(t_2-t_1)H_I(t_1;g)+H_{non}(t_2,t_1), \tag{16}$$

with

$$\begin{gathered}H_{non}(t_2,t_1)=\lim_{\Lambda\to\infty}((-i)^2H_I^{(\Lambda)}(t_2;g)H_I^{(\Lambda)}(t_1;g)+\\ +(-i)^3\int_{t_1}^{t_2}dt<0|H_I^{(\Lambda)}(t_2;g)H_I^{(\Lambda)}(t;g)H_I^{(\Lambda)}(t_1;g)|0>+i\delta(t_2-t_1)\delta m^2(\Lambda)\times\\ \times\int d^2xg(\mathbf{x}):\varphi^2(\mathbf{x},t_1):+2i\delta(t_2-t_1)E(\Lambda)\int d^2xg(\mathbf{x})).\end{gathered}$$

This is the generalized interaction operator of the renormalized φ_3^4 theory. This operator is an operator-valued distribution on Fock space, i.e. there are no problems associated with the UV divergences. Correspondingly Eq.(15) with such an operator $H_{int}(t_2,t_1)$ is ultraviolet finite and directly leads to the renormalized expression for the evolution operator and S matrix. In fact, the operator (16) is defined as a limit of the consequence of the operators $H_{int}^{(\Lambda)}(t_2,t_1)$ which are dynamically equivalent to the operators $-2i\delta(t_2-t_1)H_I^{(r)}(t_1;g)$, and generates Hamiltonian dynamics. The perturbative solution of Eq.(15) for these interaction operators are known, and are given by (11). Thus the perturbative solution of Eq.(15) with the generalized interaction operator (16) is a limit of the consequence of the operators $\tilde{S}_{r,\Lambda}(t_2,t_1;g)$ for $\Lambda\to\infty$. From this and (14) it follows that the solution of Eq.(15) with the generalized interaction operator (16) yields the same results as the renormalized theory φ_3^4. Note in this connection that the renormalization program implies that all observable quantities, such as S matrix, can be represented as a limit of these quantities calculated by using Hamiltonians with an approximate, momentum cutoff. The UV divergences manifest themselves in the fact that limiting interaction Hamiltonians are infinite and therefore physically meaningless. In fact, the consequence of the operators $H_I^{(r)}(t_1;g)$ does not converge to some finite operator. At the same time the consequence of the operators $H_{int}^{(\Lambda)}(t_2,t_1)$, that are dynamically equivalent to the operators $-2i\delta(t_2-t_1)H_I^{(r)}(t_1;g)$ for finite Λ, converges to some operator valued distribution on Fock space, that is not dynamically equivalent to any generalized interaction operator of the form (6), i.e. is nonlocal in time. Thus the dynamics of the renormalized theory is generated by the nonlocal interaction being described by the generalized interaction operator (16).

In summary, we have shown that after renormalization the dynamics of the theory φ_3^4 is governed by the generalized dynamical equation (3) with the nonlocal interaction operator (16). By solving this equation with the interaction operator (16) and removing the spartial cutoff, one can construct the finite, and Lorents invariant S matrix. Thus within the GQD this theory manifests itself as a finite theory free from UV divergences. This gives reason to suppose that such a dynamical situation takes place in any renormalizable theory, and such theories as QED and QCD may be formulated in an ultraviolet-finite way. It is hoped that the above ideas may be also applied to nonrenormalizable theories. The results of our paper also show that the GQD may open new possibilities for applying the EFT approach to nuclear physics. In fact, the essential lesson we have learned from our analysis is that the low-energy nucleon dynamics to which a EFT leads after renormalization should be governed by Eq.(3) with nonlocal generalized interaction operator that provides a natural parameterization of the nuclear forces. By using a EFT, one can construct a generalized operator of the NN interaction consistent with symmetries of QCD. This operator can then be used in Eq.(3) for describing nucleon dynamics.

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